exact expressions for the clustering and degree c
k of small world properties in the whole paramet
ese networks in the light of the present detailed a

7.23.Ge, 05.70.Ln

ION

has been devoted to
of a wide range of so-
be described as net-
nternet [3, 4, 5, 6] or
mmunities [8], food-
networks [10, 11, 12,
], in which nodes rep-
nd links the physical
y, many of these net-
operties and dynam-
nted for by classical
r, small-world prop-
ributions [17] (where
s defined as the num-
ched) seem to emerge
verning the topology
al properties imply a
d a short average dis-
nsiderable impact on
aking place on top of
e (SF) networks have
om damage (absence
20] and prone to epi-
shold) [21, 22, 23, 24].
ological properties of
l-world and scale-free
on-trivial degree cor-
Recently, an interest-
duced by Klemm and

study how clusterin
the resilience to da
networks [26, 27].

In this paper we r
model. We find an a
imal values of active
In addition, large sc
noticeable variabilit
In particular, the d
m for the general ca
topology is also susc
struction algorithm
simulations we study
the whole range of n
eters. We calculate
and connectivity cor
variability with resp
Extensive numerica
picture presented he

In the generated
small-world propert
we find a network c
number of nodes for
topology is therefo
star-shaped graphs.
one dimensional lat
properties. In part
cesses might be hea
age distance among
to a one-dimensiona
cuss the properties

et of active nodes $\mathcal{A}$,
notes the in-degree of

del is quite sensitive
3 are performed and,
e the following cases.

before step 3.

after step 3.

e solved analytically
on, after introducing
ve node has in-degree
order of steps 2 and
n-degree distribution

(2)

using $a = m$. In this
nversely proportional
s $(m+k^{\mathrm{in}})^{-1}$ and the
be $P(k) = 2m^2k^{-3}$.
n mechanism the net-
cient that approaches
limit [25].
n claimed that finite
ty distribution shows
vior. We shall see in
0 the model presents
which yields a con-
the $m \to \infty$ limit. In
pology is very sensible
ving algorithms.

with probability

$$p_d(K) = \frac{1}{1 +}$$

Each time the oldest
increases by one and
oldest node has in-d
it is not deactivate
degree $2,\ 3, \ldots K -$
creating a deactivat
the probability that
in $K-2$ steps and

$$\tilde{P}(K) =$$

$$=$$

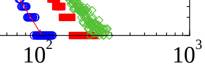
where $\Gamma(x)$ is the s
the other hand, eve
deactivated, the oth
Hence, in the $K -$
generation of a node
in-degree 1 are crea
with in-degree 1 cre

$$\tilde{P}_1 = \sum_{K=}^{\infty}$$

Therefore, the in-de

$$P(k^{\mathrm{in}}) = C$$

where C is a consta
condition $\sum_{k^{\mathrm{in}}} P(k^{\mathrm{i}}$



A for $a = m$, network
of m. The continuous
$= 2$ given by Eq. (10).
nent γ as a function of
s.

evious expression us-
ain that the in-degree
behavior

$$= 2 + a. \quad (9)$$

l nodes is m then the
ut-degree) is $m + k^{\text{in}}$
on shifted by m. For
e degree distribution

$$k = 3 \qquad (10)$$
$$\overline{3)} \qquad k > 3$$

bution obtained from
or $a = m$. For $m = 2$
d agreement with the
0), with a power law
. In the limiting case
predicts the exponent
er bound. Hence,

$$3 < \gamma \leq 4 \quad (11)$$

$$p_a(K) \quad 2a$$

The probability tha
it has degree K is g

$$\tilde{P}(K) =$$
$$=$$

In the process of cr
nodes of in-degree
number of nodes wi

$$\tilde{P}_0 = \sum_{K=}^{\infty}$$

Thus, the analytic
degree distribution

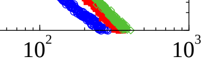
$$P(k^{\text{in}}) = C$$

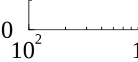
with the normalizat

$$C = \tilde{P}$$

From here follows th
tion (where $k = m$

$$P(k) = \begin{cases} \\ \frac{\Gamma(2}{\Gamma(a} \end{cases}$$





B for $a = m$, network
of m. The inset shows
on of m obtained from

ow a power law decay
ntinuously increasing
g that for $m < 10$ the
ngly differs from the

$n = 1$ the analytic so-
readily seen from the
In fact, the solution
n the thermodynamic
weight of the nodes
respect to the nodes
ngularity is rooted in
n exponent $\gamma = -2$,
hermodynamic limit,
model B has average
ily implies that there
the network size N in
$= 1$, dependence that
solution since we are
work limit. We can
l form of the degree
mposed by N nodes,
y k_c, such that there
at k_c. Assuming that
the same functional
$a = 1$

$= 1$

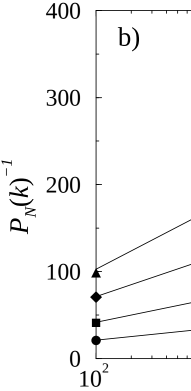


FIG. 3: Scaling of th
model with $a = m =$
size N, for (a) $k =$
least-squares fits to th
$P_N(k)^{-1} \sim \ln N$ in (b

proximation that re

$$C_1 = 1 - \frac{2\ln(3}{\ln\left(\frac{1+}{2}\right.}$$

For finite SF netwo
$k^{-\gamma}$, the maximum
nodes as $k_c \sim N^{1/(}$
$k_c \sim N$, and thus, fo

$$1 - C_1$$

tion has a divergent

wn that the deactiva-
order in which steps 2
e distributions with a
depending on the or-
rather sensible to the
ge of variation. This
ous works where this
27], prompting that
n those works should

EFFICIENT

ribution and compute
function of the node
we can perform an
of a and m and for
ompute the clustering
etwork as undirected
al degree of the node

node i is defined by

(24)

ween the neighbors of
imum possible value
model new edges are
and the added node.
active nodes and in-
oreover, all the active
time we add a node
increases by one and

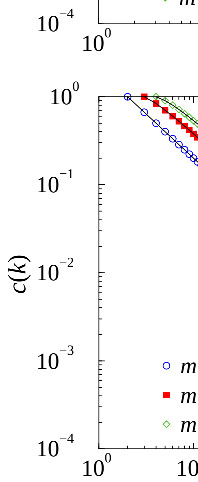


FIG. 4: Clustering co
for different values of
merical simulations o
network size $N = 10^5$
the analytical solution

i was created. Besi
degree m, thus $e_i(0)$
1. Integrating Eq.
substituting the res
that $t = k_i - m$, we

$$c(k) = \frac{m}{k}$$

$$= \frac{2(}{}$$

where the last expr
some algebraic mani

s of the neighbors of
he quantity $k_{nn,i}$ does
degree of the node i.
relations are present.
he degree of the node
ed. In particular, we
ation. In the first sit-
ty will connect more
s; a property referred
e opposite side, it is
ng"; i.e. highly con-
ed to nodes with low

n the node is added
$_{\mathcal{A}}$, where $\langle k\rangle_{\mathcal{A}}$ is the
ors. Then, if the node
eighbor of the $m-1$
y time a new node is
ncreases by $(m-1)+$
the remaining $m-1$
e and the m because
nce

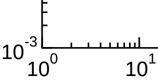
$$), \qquad (28)$$

this equation, taking
$D_i(0) = m\langle k\rangle_{\mathcal{A}}$ and
hat

$$+ m\langle k\rangle_{\mathcal{A}} \qquad (29)$$

Now, when an active
emains fixed but the
will still increase until
he infinite time limit,

$$(30)$$



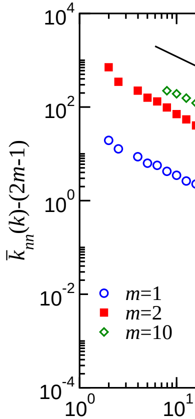
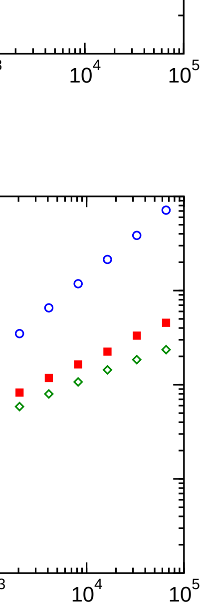


FIG. 5: Average near
degree k for different
from numerical simula
up to a network size
tions. The continuou
dependency $\bar{k}_{nn}(k) -$

time average of $\langle k\rangle$
nectivity of any dea
($m = 2$ for model
gree of an active no
the degree of the re
maining nodes have
with $m = 2$, and d
dependently of the
Therefore in this ca



ree as a function of the
f m. The points were
of (a) model A and (b)
^{5}, averaging over 1000

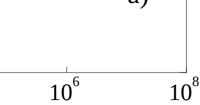
$1/k$ even for $m \neq 2$,
he case of model B,
$m = 1$ and $m = 10$
values of m. Thus,
n the active node de-
for intermediate val-
we find that correla-
e of “disassortative”
s are preferably con-

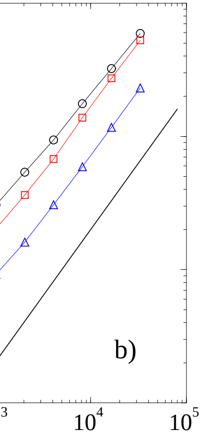
limit $\gamma = 3$ for m
mically with N. O
$2 \leq \gamma < 3$, $\langle \bar{k}_{nn} \rangle_N$
law. This implies th
and that in the the
form first the limit N
connectivity curve i
larger values. This
nearest neighbor con
quantity since the k
after the $N \to \infty$ li
N is related to a ge
diverging connectivi
the detailed balance

VI. DIAMET

Another fundame
networks is identifie
length among node
minimum path betw
imum number of in
versed to go from n
path length $\langle d \rangle$ is th
tance averaged over
network. Similarly,
the largest among t
nodes in the networ

While regular ne
tices) have a diame
inverse of the Eucli
works show striking
erage one can go fr
the system by passi
intermediate nodes





d the average shortest
rent values of m. The
ke of clarity, the curve
factor 5.

nplies that the topol-
proaching those of a
ords, the deactivation
properties.
sentation of the deac-
in Fig. 8 the illustra-
del B and $a = m = 3$.

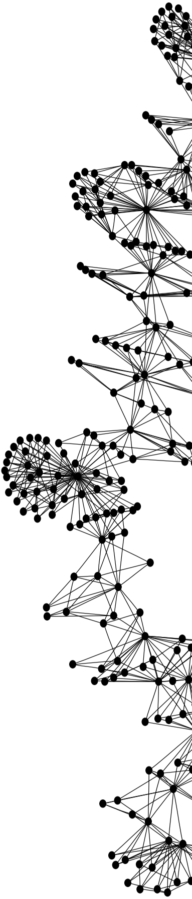
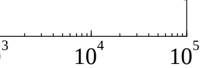


FIG. 8: Illustration o
deactivation model B
The linear topology w
is evident.



. random walker on the
ne dimensional lattice,
$V = 10^5$ nodes, as well
9 with 6301 nodes.

all discuss in the last
properties might have
properties of the net-

ONCLUSIONS

vided a detailed anal-
roduced in Ref. [25].
being very sensible to
the model and slight
n. The most striking
n is depending on the
usly active nodes m
e. when the deactiva-
les' total degree. The
pproaching the value
properties of networks
$1 \leq m \leq 10$. Along
n previous works, we
sting degree correla-
ind marked disassor-
connected nodes link
es. The analytical ex-
is obtained and re-
trikingly, the SF and

the percolation tra
are extremely robus
natural question is t
properties of SF net
general results obta
this reason, several
fect of such correlat
ring on these netwo
Ref. [26] it has been
threshold in the cas
B.

The presence of a
model has been trac
cient and the finite
connectivity of the l
we have shown here
connectivity in the
size. What appears
erties of spreading
ear structure, with
In a coarse grained
dominated by the
chain. In order to
a standard random
Fig. 9 we plot the m
dom walker, $\langle R^2(t)$
brackets denote an
random walk on 250
fusive system, as wo
lattice, we would ex
the deactivation m
sive behavior, with
as $\langle R^2(t) \rangle^{1/2} \sim t^{0.4}$
on the deactivation
expected from its n
ysis of spreading an

w the introduction of
s the usual absence of
spective, it would be

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